Math 62 : Word problems for Final Exam

Math 72 : 3.2 Word problems

Lesson 2

Math 70 B 2.4 & 2.5 Word Problems

(We are skipping forward to the first sections that can be done without a graphing calculator.)

2.4 Applications with Percent

Objectives

1) Write percent as decimal.
2) Write decimal as percent.
3) Solve basic percent problems.

2.5 Problem Solving

Objectives

Solve problems using

1) direct translation
2) consecutive integers, consecutive even integers, or consecutive odd integers
3) geometry formulas
    a. perimeter
    b. area of a rectangle
    c. angles of a triangle
    d. complementary or supplementary angles
4) money, including
    a. fixed and variable costs
    b. break-even analysis of cost and revenue
5) percent increase or percent decrease
6) uniform motion (D=RT)

Examples & Practice Problems

Define a variable, write an algebraic equation using that variable, and solve the equation to answer the question. If the question includes units, always include them in your answer.

1) Six times some number added to 8 amounts to 4 added to the product of 4 and the number.

2) The sum of three consecutive even integers is 228. Find the integers.

3) The perimeter of a cross section of a "two by four" piece of lumber is 10 inches. The length is 2 inches longer than the width. Find the actual dimensions of the cross section of a two-by-four.

4) The length of a rectangle is two feet more than the width. The area is 131.25 square feet. Find the dimensions of the rectangle.

5) Find the measures of the angles of a triangle if the measure of the first angle is three times the measure of the second angle and the third angle is 25° more than the second angle.

6) Angle A and angle B are complementary angles and angle A is 14° more than three times angle B. Find the measures of angle A and angle B.

7) Angle A and angle B are supplementary angles and angle A is 35° less than four times angle B. Find the measures of angle A ad angle B.

8) Fine Line Trucks rents an 18-ft truck for $42 plus 35 cents per mile. Judy needs a truck for one day to deliver a shipment. How far can she drive and stay within a budget of $70?

9) The cost C to produce x number of tennis rackets is $C = 170 + 13x$. The tennis rackets are sold wholesale for $18 each, so revenue R is given by $R = 18x$. Find how many tennis rackets the manufacturer needs to produce and sell to break even.

10) Find 13% of 5000.

11) The population of a town increased by 200%, to 100 people. Find the original population of the town.

12) A diamond ring sold for $2875.50 including tax. If the tax rate where the diamond was purchased is 6.5%, find the price of the ring before the tax was added. (Round to the nearest cent, if necessary.)

13) During a stock market "correction", Eva's retirement funds lost 8% of their value. After the crash, Eva had $180,000 in her retirement account. How much was in the account before the crash?

14) Rafaela ran for 20 minutes and then walked for 10 minutes. Her running rate was 250 feet per minute faster than her walking rate. If she ran and walked a total of 15,500 feet, how fast did she run?

15) Antonio drove for 2 hours in a snowstorm and then for 5 hours more in clear weather. He drove half as fast through the snow as he did in the clear weather. If he drove 240 more miles in clear weather than he did in the snow, how fast did he drive through the snow?

16) Emory rode their bicycle at a rate of 15 mph and then slowed to 10 mph. They rode 30 minutes longer at 15 mph than they did at 10 mph. If they traveled a total of 25 miles, how long did they ride at the faster rate? [Caution: Be careful the units match!]

PHRASES MEANING ADDITION

Phrase	*Translation*
"**the sum of** 3 and a number"	$3 + x$
"**the total of** a number and 50"	$x + 50$
"**8 more than** a number"	$x + 8$
"a number **increased by** 22"	$x + 22$
"a number **plus** 10"	$x + 10$
"one number **added to** another"	$x + y$

PHRASES MEANING SUBTRACTION

Phrase	*Translation*
"4 **subtracted from** a number"	$x - 4$
"**the difference of** 4 and a number"	$4 - x$
"4 **decreased by** a number"	$4 - x$
"4 **less than** a number"	$x - 4$
"4 **fewer than** a number"	$x - 4$
"4 **minus** a number"	$4 - x$
"4 **reduced by** a number"	$4 - x$
"4 **less** a number"	$4 - x$

PHRASES MEANING MULTIPLICATION

Phrase	*Translation*
"a number **multiplied by** 5"	$5 \cdot x$ or simply $5x$

(We usually write variable factors last, so we write $5x$ rather than $x5$.)

"**the product of** 3 and a number"	$3x$
"one number **times** another"	xy
"$\frac{2}{3}$ **of** a number"	$\frac{2}{3}x$

Special multiplication phrases

"**twice** a number"	$2x$
"**double** a number," "a number **doubled**"	$2x$
"**triple** a number," "a number **tripled**"	$3x$

PHRASES MEANING DIVISION

Phrase	*Translation*
"a number **divided by** 8"	$x \div 8$ or $\frac{x}{8}$
"the **quotient** of a number and 8"	$x \div 8$ or $\frac{x}{8}$
"the **ratio of** a number to 8"	$x \div 8$ or $\frac{x}{8}$
"a number **divided into** 8"	$x\overline{)8}$ or $8 \div x$

Special division phrase

"the **reciprocal** of a number"	$\frac{1}{x}$

KEY VERBS AND VERB PHRASES MEANING "EQUALS"

Phrase	*Translation*
"**equals**" or "**is equal to**"	$=$
"**is**" or "**are**" or "**was**"	$=$
"**gives**" or "**gives us**"	$=$
"**gives a result of**" or "**results in**"	$=$
"**leaves us**" or "**leaves**" or "**left**"	$=$
"**is the same as**"	$=$

PHRASES MEANING POWERS OR ROOTS

Phrase	*Translation*
"**the square** of a number"	x^2
"**the cube** of a number"	x^3
"**a number raised to the fourth power**"	x^4
"**the sixth power** of a number"	x^6
"**The square root** of a number"	$\sqrt{x}$

Math 45: Useful Math Terminology

Operation: add, subtract, multiply, divide, exponent, radical, absolute value, etc.

Order of Operations: Rules for which part of a calculation to do first, and then what order to proceed

Variable: A letter used to represent any or a specific number, often unknown

Expression: A combination of variables and numbers with no equal sign, though operations are permitted

Substitute: Replace a variable or an expression by a variable or expression which is known to be equal

Evaluate: Give a number answer as a result, often by substituting given values for a variable then performing any operations

Terms: Parts of an expression that are separated by addition (rewrite subtraction as addition)

Distribute: Multiply a single term by all terms within parentheses that follow or precede it

Factors: Parts of a term that are separated by multiplication (rewrite division as multiplication)

Factor: Find the prime factors (numbers or polynomials) which multiply to give the original expression

Reduce: Put a fraction in lowest terms. A reduced fraction has no common factors, decimals or fractions within fractions

Constant: An expression which is a number without variables

Coefficient: A number, especially if multiplied times a variable or product of variables or quotient of variables

Equation: Two expressions separated by an equal sign

Inequality: (a) One of the four symbols $<, >, \leq, \geq$
(b) Two expressions separated by an inequality symbol

Solve: Find all solutions, meaning the values for a variable which make an equation or inequality true. Solution may be a single value, list, set, interval, or a graph.

Isolate: Perform operations to both sides of an equation or inequaltiy so the isolated item is alone on one side

Set equal to zero: (a) Set an expression equal to zero: write the expression, an equal sign, and a zero when finding the x-intercepts or zeros of the expression viewed as a function.
(b) Set an equation equal to zero: Rearrange terms by adding or subtracting the same term from both sides of the equation until one side is zero.

Rational: A number or expression which is or can be written as a fraction if integers or a fraction of polynomials

Simplify: Final form for an answer. Reduce fractions, distribute, combine like terms, evaluate operations. If simplifying a rational expression, factor and divide out common factors, then leave final answer factored.

Interval: A set of all values between two given endpoints. Notation specifies if endpoints are included.

Ordered Pair or Point: A pair of numbers in parentheses, separated by a comma, denoting the x- and y-coordinates on a rectangular coordinate graph

Geometry Formulas

DEFINITIONS

The **perimeter** is the sum of the lengths of all the sides of a figure.

The **area** is the amount of space enclosed by a two-dimensional figure measured in units squared.

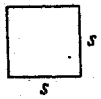
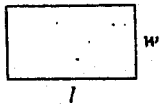
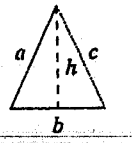
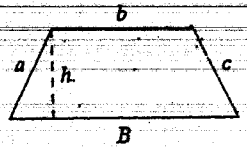
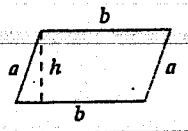
The **surface area** of a solid is the sum of the areas of the surfaces of a three-dimensional figure.

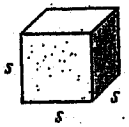
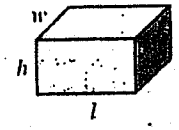
The **volume** is the amount of space occupied by a three-dimensional figure measured in units cubed.

The **radius** r of a circle is the line segment that extends from the center of the circle to any point on the circle.

The **diameter** of a circle is any line segment that extends from one point on the circle through the center to a second point on the circle. The diameter is two times the length of the radius, $d = 2r$.

In circles, we use the term **circumference** to mean the perimeter.

Plane Figures		Formulas
Square		Area: $A = s^2$ Perimeter: $P = 4s$
Rectangle		Area: $A = lw$ Perimeter: $P = 2l + 2w$
Triangle		Area: $A = \frac{1}{2}bh$ Perimeter: $P = a + b + c$
Trapezoid		Area: $A = \frac{1}{2}h(B + b)$ Perimeter: $P = a + b + c + B$
Parallelogram		Area: $A = bh$ Perimeter: $P = 2a + 2b$
Circle		Area: $A = \pi r^2$ Circumference: $C = 2\pi r = \pi d$

Solids		Formulas
Cube		Volume: $V = s^3$ Surface Area: $S = 6s^2$
Rectangular Solid		Volume: $V = lwh$ Surface Area: $S = 2lw + 2lh + 2wh$
Sphere		Volume: $V = \frac{4}{3}\pi r^3$ Surface Area: $S = 4\pi r^2$
Right Circular Cylinder		Volume: $V = \pi r^2 h$ Surface Area: $S = 2\pi r^2 + 2\pi rh$
Cone		Volume: $V = \frac{1}{3}\pi r^2 h$

Math 70 B 2.4 & 2.5

① Six times some number added to 8 amounts to

translate: 6 · x + 8 =

4 added to the product of 4 and the number.

translate: 4 + 4 · x

Resulting equation: $6x + 8 = 4 + 4x$

collect variables: $-4x \quad -4x$

$2x + 8 = 4$

isolate variable: $-8 \quad -8$

$2x = -4$

$\frac{2x}{2} = \frac{-4}{2}$

$\boxed{x = -2}$

② The sum of 3 consecutive even integers is 228.

[ ] + [ ] + [ ] = 228

1st 2nd 3rd

consecutive: in a row
even: evenly divisible by 2
integer: no fraction or decimal part

ex: 50, 52, 54 or 66, 68, 70 (+2, +4)

$\boxed{x} + \boxed{x+2} + \boxed{x+4} = 228$

M70

$x + x + 2 + x + 4 = 228$

$3x + 6 = 228$ combine like terms

$-6 \quad -6$ isolate x

$3x = 222$

$\frac{3x}{3} = \frac{222}{3}$

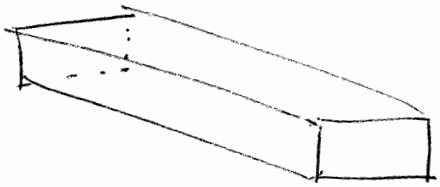
$x = 74$

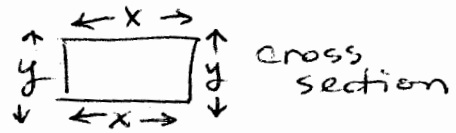
$x + 2 = 74 + 2 = 76$

$x + 4 = 74 + 4 = 78$

74, 76, 78

(3) The perimeter of a cross-section of a "two by four" piece of lumber is 10 inches





perimeter: add the lengths of sides, going around the edge:

$x + y + x + y = 10$

$\frac{2x}{2} + \frac{2y}{2} = \frac{10}{2}$ or $\underline{(x) + y = 5}$

The length is 2" longer than width. $\underline{x = (y + 2)}$
Find actual dimensions.

substitute for x: in 1st (perimeter) equation $(y+2) + y = 5$

$2y + 2 = 5$

$-2 \quad -2$

$2y = 3$

M.to

$$\frac{2y}{2} = \frac{3}{2}$$

$y = \frac{3}{2}$ in or 1.5 in

$x = y + 2 \Rightarrow x = \frac{3}{2} + 2$ or $1.5 + 2$

$= \frac{3}{2} + \frac{4}{2}$ $= 3.5$ in

$= \frac{7}{2}$ in

dimensions: 1.5 in x 3.5 in

or $\frac{3}{2}$ in x $\frac{7}{2}$ in

(4) The length of a rectangle is 2 feet more than the width. The area is 131.25 square feet. Find the dimensions of the rectangle

Length = 2 + Width $\Rightarrow$ $L = (2 + W)$

Area = $L \cdot W$ $\Rightarrow$ $131.25 = (L)W$

Substitute for L: $131.25 = (2+W) \cdot W$

distribute $131.25 = 2W + W^2$

↑ uh-oh! exponent 2 is
- called "quadratic"
- is not linear
- cannot be solved by isolating the variable

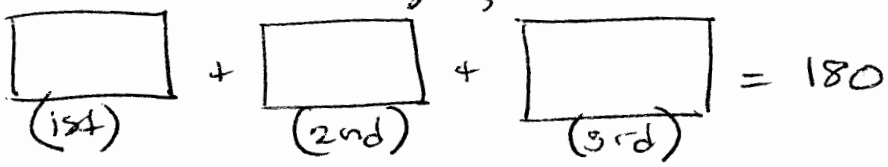
STOP!

We won't be able to solve this question until
a) we can use a GC
or b) we learn quadratics (chapter 11)

M70

(5) Find the measures of the angles of a triangle if

3 angles, add to 180°



...The measure of the first angle is 3 times the measure of the second

(1st) = 3 · (2nd)

...and the third angle is 25° more than the second angle

(3rd) = 25 + (2nd)

Notice that both (1st) and (3rd) are equal to expressions containing (2nd).

⇒ They can be written in terms of 2nd.

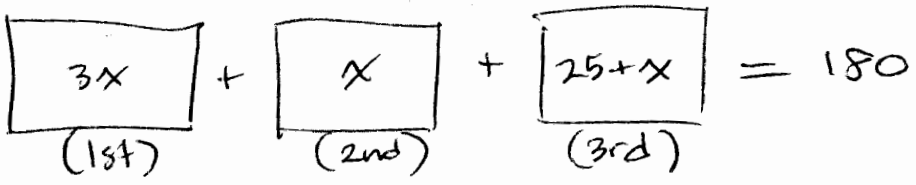
⇒ This means that (2nd) is an excellent choice for a variable

Let x = 2nd angle ⇒ (2nd) = x

(1st) = $3x$

(3rd) = $25+x$

Substitute these into the boxes we wrote at first:



$3x + x + 25 + x = 180$ remove boxes

$5x + 25 = 180$ combine like terms on LHS

$-25 \quad -25$

$5x = 155$ isolate x

M70

$\frac{5x}{5} = \frac{155}{5}$ isolate x

$x = 31^\circ$

$3x = 3(31) = 93^\circ$

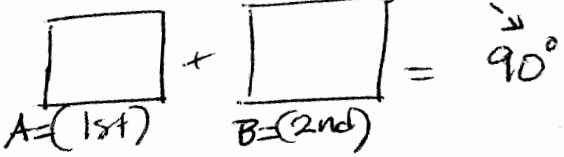
$25 + x = 31 + 25 = 56^\circ$

check: $31 + 93 + 56 = 180^\circ$ ✓

answer: $\boxed{31^\circ, 93^\circ, 56^\circ}$

(6) Angles A and B are complementary angles

two angles => two boxes



... and angle A is 14° more than three times angle B.

$A = 14 + 3B$

$\left.\begin{array}{l} (A) + B = 90 \\ A = (14 + 3B) \end{array}\right\}$ substitute

$(14 + 3B) + B = 90$

$14 + 4B = 90$ combine like terms

$-14 \quad -14$

$\frac{4B}{4} = \frac{76}{4}$ isolate x (B in this case)

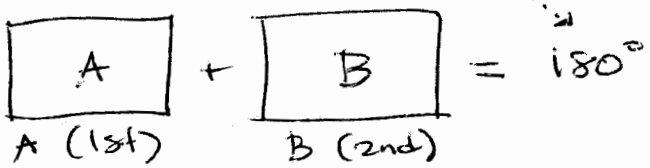
$\boxed{B = 19^\circ}$

$A = 14 + 3B$ or $A + B = 90$

$A = 14 + 3(19)$ $\quad A + 19 = 90$ $\Rightarrow \boxed{A = 71^\circ}$

Math 70

(7) Angle A and angle B are supplementary angles...



...and angle A is 35° less than 4 times angle B.

A = 4·B − 35

$(A) + B = 180$
$A = (4B - 35)$
} substitute

less than means
- subtract
- in the opposite order of their appearance in the sentence

$(4B - 35) + B = 180$

$5B - 35 = 180$ combine like terms

$+35 \quad +35$ isolate variable B

$\frac{5B}{5} = \frac{215}{5}$

$\boxed{B = 43°}$

$A = 4B - 35$ or $A + B = 180$

$A = 4(43) - 35$ $A + 43 = 180$

$\boxed{A = 137°}$

Math 70

(8) Fine-line trucks rents an 18-foot truck ...

↓ irrelevant.

... for $42 plus 35 cents per mile.

↓ pay only once

add | convert to dollars | multiply by # of miles

cost → 42 + .35 (# miles)

... Judy needs a truck for one day to deliver a shipment. How far can she drive and stay within a budget of $70?

↓ distance, measured in miles

↓ less than or equal to $70.

- This is the question! Let x = # miles.

cost → $42 + .35x \leq 70$.

or just $42 + .35x = 70$ for maximum number of miles

$-42 \qquad -42$

$$\frac{.35x}{.35} = \frac{28}{.35}$$ isolate x

$x =$ **60 miles** * always include units

(9) The cost C to produce x tennis rackets is $C = 170 + 13x$.

↓ we are manufacturers! Imagine that!

... The tennis rackets are sold wholesale for $18 each

↓ reword "for $18 per racket"

↓ mult by # of rackets.

... So revenue is given by $R = 18x$

↳ cash into the cash register — no bills paid yet!

Math 70

$R = 18x$

↑ multiply by x = # rackets, same as (#8) "per".

... Find how many tennis rackets to produce to break-even.

costs = revenues

pay bills — enough money

After the break-even point our business makes money.

Costs = Revenues

$170 + 13x = 18x$

$-13x \quad -13x$ collect variables on same side

$\frac{170}{5} = \frac{5x}{5}$ isolate x

$34 = x$

Sell **34 rackets** to break even

* always include units!

(10) Find 13% of 5000. ← base

13% — a portion; of means multiply

$.13 = \frac{13}{100}$, as a fraction or decimal

Basic percent
$a = p \cdot b$

$a = (.13)(5000)$

$a = \boxed{650}$

M70

(11) The population increased by 200%, to 100 people. Find the original population of the town.

Key question: 200% of what?

either 200% of unknown starting population $\Rightarrow 2.00x$

OR 200% of ending population 100 $\Rightarrow 2(100) = 200$?!?

In percent increase OR percent decrease problems, it is always $(\text{percent}) \times \left(\begin{matrix}\text{earlier} \\ \text{base} \\ \text{value}\end{matrix}\right)$

So $2x$ is correct and 200 is incorrect.

Percent increase equation

$b + p \cdot b = \text{future}$ ← our question says "increase"

Percent decrease equation

$b - p \cdot b = \text{future}$

b = earlier base value, unknown x

$$x + 2.00x = 100$$

$$3x = 100$$

$$x = \frac{100}{3}$$

$x \simeq 33\frac{1}{3} \simeq$ about **33 people**

$\frac{1}{3}$ person doesn't make sense so round to give meaningful answer.

Math 70

(12) A diamond sold for $2875.50 including tax.

↓

The money listed is price + tax

... If the tax rate is 6.5%, find price before tax.

↓ ↓

.065 as decimal earlier base b is unknown

Tax is added to the price ⇒ percent increase

$b + p \cdot b = 2875.50$

$b + .065b = 2875.50$

$1b + .065b = 2875.50$

$(1 + .065)b = 2875.50$

$1.065b = 2875.50$

$\frac{1.065b}{1.065} = \frac{2875.5}{1.065}$

$b = \boxed{\$2700}$

Combine like terms

easier examples:

- $b + b = 2b$
- $.1b + .1b = .2b$
- $b + .1b = (1 + .1)b = 1.1b$

isolate variable

Math 70

(13) During a stock market correction, Eva's

↓

The value of stocks "crashed" or went down, decreased. ⇒ percent decrease

... retirement funds lost 8% of their value

↓ .08 as decimal; ↓ multiply; ↓ original value, earlier in time

... After the crash, Eva had $180,000 in her account

↓

$180,000 is the ending or final amount

... How much was in the account before the crash?

→ find original ← base amount = b.

percent decrease equation

$b - p \cdot b = \text{future}$

$b - .08b = 180000$

$(1 - .08)\, b = 180000$

$.92b = 180000$

$\frac{.92b}{.92} = \frac{180000}{.92}$

combine like term

$\begin{cases} 2b - b = b \\ 1b - .1b = (1 - .1)b \\ \qquad = .9b \end{cases}$

easier examples

$b = 195652.1739$

$\boxed{\$195{,}652.17}$

round to nearest hundredth (cent) to make sense for money

Math 70

(14) Rafael ran 20 min then walked 10 min. His running
↓ time — ↓ also time

... rate was 250 ft per min faster than his walking rate.
↓ Keyword "rate" — ↓ add 250 ft/min to his walking rate

... If he ran and walked a total of 15500 feet, how
↓ add together — ↓ distance

... fast did he run?
↓ find rate for running. ⇒ x = rate for running

"Distance", "Rate", "Time" ⟹ key words mean $D = R \cdot T$

	D	= R	· T
run	?	x	20 min
walk	?	250 ft/min slower than x	10 min

total: 15500

To make a table, spread out the formula, make one line for running and one line for walking.

Note: we could set this up with x = rate for walking and x + 250 = rate for running, and do an extra step at the end.

"Distance = rate times time" is true for walking and it's true for running. Mult R·T and put the result in D.

Math 70

$D = R \cdot T$

$20x$	x	20
$10(x-250)$	$x-250$	10

total 15500

"total distance" means "add the distances"

$$20x + 10(x-250) = 15500$$

distribute $\quad 20x + 10x - 2500 = 15500$

combine $\quad 30x - 2500 = 15500$

isolate $\quad +2500 \quad +2500$

$$\frac{30x}{30} = \frac{18000}{30}$$

$$x = \boxed{600 \text{ ft/min}}$$

Realistic? Let's convert to mph:

$$600 \frac{\text{ft}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 6.8 \text{ miles per hour.}$$

Math 70

(15) Antonio drove 2 hrs in snowstorm and 5 hours in clear

2 hrs → time

5 hours → time

... He drove half as fast through snow as in clear.

$\frac{1}{2}$ · rate

... He drove 240 more miles in clear than in snow

add 240 miles → to snow distance

... How fast did he drive through snow?

find rate → in snow ⇒ x = rate in snow

Distance, Rate, Time ⇒ $D = R \cdot T$

$$D = R \cdot T$$

	D	R	T
snow	D_{snow}	x	2
clear	240 miles + D_{snow}	twice as fast as x	5

If (snow rate) is half of (clear rate)
then (clear rate) is twice (snow rate)

ex: snow clear: 40 and 80; clear snow: 80 and 40

$$D = R \quad T$$

less →	$2x$	x	2
240 mi more →	$5 \cdot 2x$	$2x$	5

Math 70

$10x$ (is 240 miles more than) $2x$

$10x$ ↑ bigger

$2x$ ↑ smaller

$2x + 240 = 10x$

$2x + 240$ ↑ add to smaller expression

$= 10x$ ↑ larger expression

= to make equal to

$2x + 240 = 10x$ collect x's same side

$-2x \quad -2x$

$\frac{240}{8} = \frac{8x}{8}$ isolate x

$30 = x$

$x = 30$ mph in snow

Math 70

(16) Emery rode their bicycle at a rate of 15 mph, then

↓ key word

... slowed to 10 mph. They ode 30 min longer at 15mph

↓ time ↓ add 30 min to other time

... than they did at 10mph. If they traveled a total of 25 mile

↓ add ... → distances

... how long did they ride at the faster rate?

↓ time ← key word ↳ x = faster time

... Caution: Do units match??

Distance, Rate, Time ⇒ $D = R \cdot T$

$D = R \cdot T$

	15 mph	x
	10 mph	$x - 30$ min

longer = bigger # (minutes!)

shorter = smaller # subtract

total 25 miles

miles per hour means $\frac{\text{miles}}{\text{hrs}}$

and implies time is in hours.

$$30 \text{ min} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{30}{60} = \frac{1}{2} \text{hr} = .5 \text{ hr}$$

$D = R \cdot T$

$15x$	15 mph	x
$10(x+.5)$	10 mph	$x - .5$

total: 25

Math 70

add the distances to get the total:

$$15x + 10(x - .5) = 25$$

$15x + 10x - 5 = 25$ distribute

$25x - 5 = 25$ combine

$+5 \quad +5$ isolate x

$$\frac{25x}{25} = \frac{30}{25}$$

$$\boxed{x = 1.2 \text{ hr}}$$

Does this make sense?

$$1.2 \text{ hr} \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) = \boxed{72 \text{ min}}$$

yes, that's reasonable.

(11)

1.6.37 The occupations of biomedical engineers, skin care specialists, and physician assistants are among the 10 with the largest growth from 2008 to 2018. The number of physician assistant jobs will grow 2 thousand less than 3 times the number of biomedical engineer jobs. The number of skin care specialist jobs will grow 15 thousand more than half the number of biomedical engineer jobs. If the total growth of these three jobs is predicted to be 58 thousand, find the predicted growth of each job.

Define variables

Translate this sentence #1

sentence #2

translate #3

The predicted growth of biomedical engineers is ☐ thousand. (Simplify your answer.)

The predicted growth of skin care specialists is ☐ thousand. (Simplify your answer.)

The predicted growth of physician assistants is ☐ thousand. (Simplify your answer.)

growth # Biomedical engineers = B
growth # Skin care specialists = S
growth # physician assistants = P

} 3 unknowns ⇒ need 3 equations

sentence #1

PA will grow 2000 less than 3 times BE
P = 3B − 2000 (subtract, reverse order)

$P = 3B - 2000$ an equation! hurray! ☺

sentence #2

SCS will grow 15 thousand more than half the BME.
S = 15000 + $\frac{1}{2}B$

another equation! ☺

sentence #3

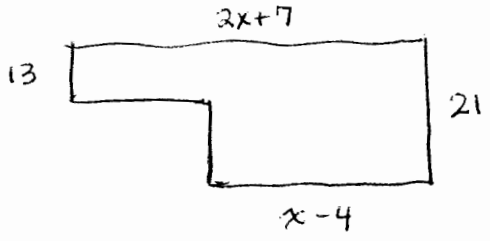
Total is predicted to be 58 thousand
add up all 3

$B + S + P = 58000$ another equation! ☺

Math 70 1.6

Extras

(12) a) Write an expression for the perimeter of this floor plan, and simplify result completely.

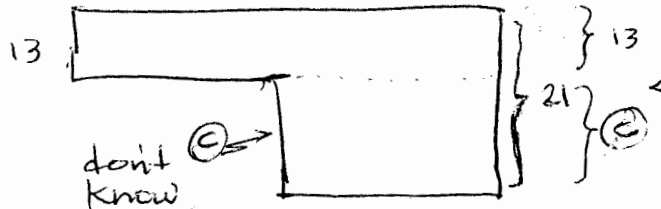


Notice! Two sides have not been labelled. ☹

Perimeter = sum of lengths of sides

We assume floor plans have right angles, so this makes the three vertical lines parallel and the three horizontal lines parallel.

Method 1:



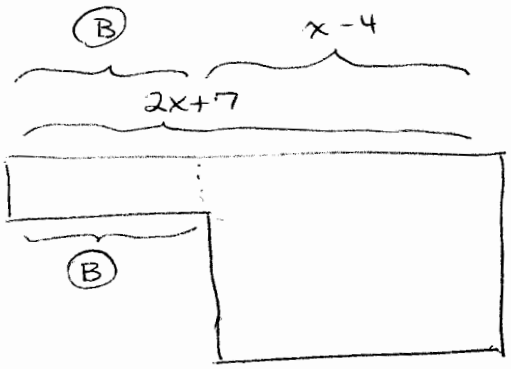
← If we mentally slide all the vertical lines to the right, we see that they add up

$C + 13 = 21$

Solve for C

$C = 21 - 13$

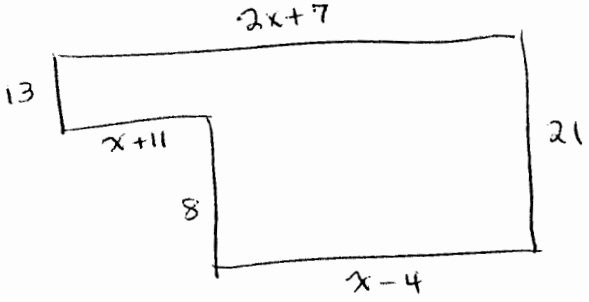
$C = 8$



← If we mentally slide all the horizontal lines to the top, we see that they add up

$B + (x-4) = (2x+7)$

or $B + x - 4 = 2x + 7$



Solve for B:

$B = 2x + 7 - x + 4$

$B = x + 11$

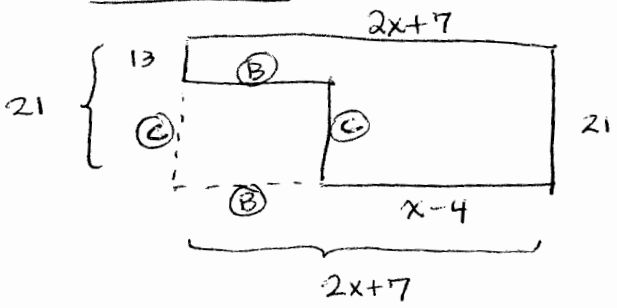
Now we know all the sides.

Math 70 1.6

Perimeter = add up all the sides

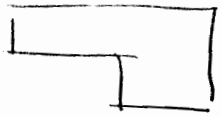
$$P = (2x+7) + 21 + (x-4) + 8 + (x+11) + 13$$
$$= 2x + x + x + 7 + 21 - 4 + 8 + 11 + 13$$
$$= \boxed{4x + 56}$$

Method 2:



Mentally slide the side (C) to the left. When added to 13, we get 21 from the opposite side.

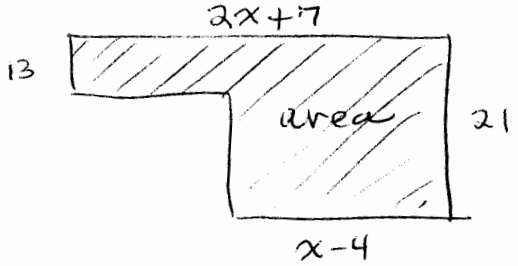
Mentally slide the side (B) to the bottom. When added to (x-4), we get (2x+7) from the opposite side.

The perimeter of  is equivalent to the perimeter of  !

$$P = 2(2x+7) + 2(21)$$
$$P = 4x + 14 + 42$$
$$\boxed{P = 4x + 56}$$

P = 2L + 2W
Perimeter of rectangle

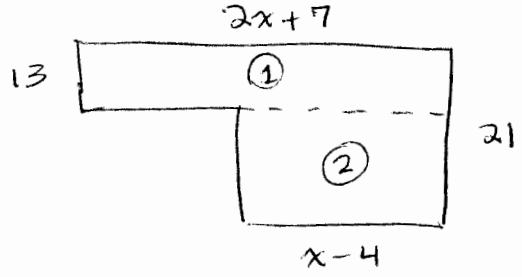
b) Write an expression for the area of the same figure

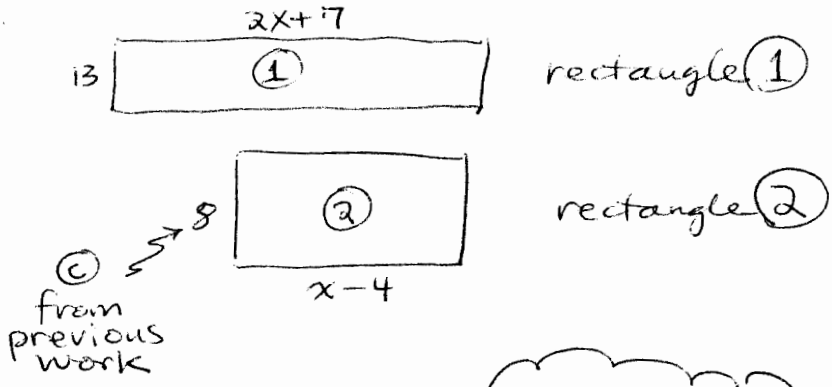


Area = amount of space inside figure

Math 70 1.6

Method 1. Divide into two regions using a horizontal lin



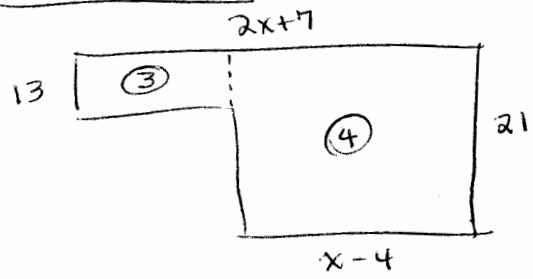


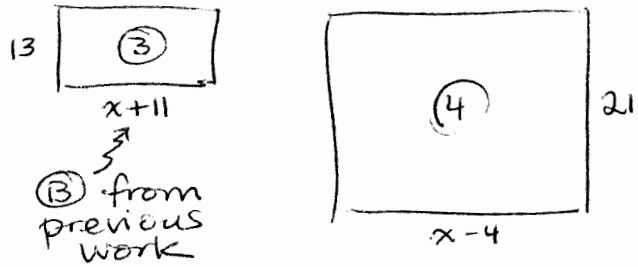
Total area = area of rectangle ① + area of rectangle ②

A = L·W area of a rectangle

$$\text{area} = 13(2x+7) + 8(x-4)$$
$$= 26x + 91 + 8x - 32$$
$$\boxed{A = 34x + 59}$$

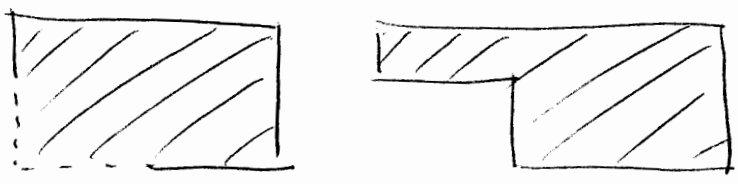
Method 2: Divide into two regions using a vertical line.





$$\text{area} = 13(x+11) + 21(x-4)$$
$$= 13x + 143 + 21x - 84$$
$$\boxed{A = 34x + 59}$$

Question: Would we get the correct area by drawing a single rectangle as for the Perimeter Method 2?



No — the space inside (area) would be larger.

Math 70 1.6

c) If the perimeter is 80 ft, find x, A, and all sides.

$P = 4x + 56 = 80$ ignore P, solve for x.

$4x + 56 = 80$

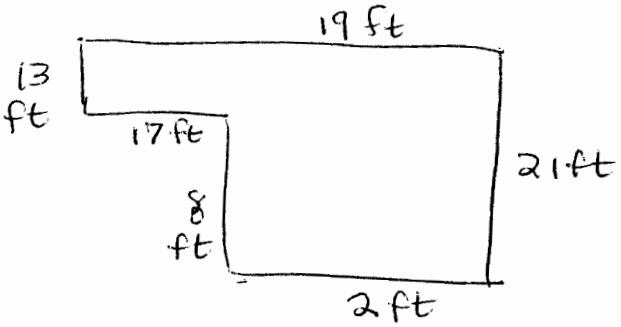
$4x = 24$

$\boxed{x = 6 \text{ ft}}$

$A = 34x + 59$ Substitute $x = 6$.

$A = 34(6) + 59$

$\boxed{A = 263 \text{ ft}^2}$



$2x + 7$ Subst $x = 6$

$2(6) + 7$

$= 12 + 7$

$= 19$ ft

$x - 4$ Subst $x = 6$

$6 - 4$

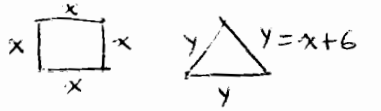
$= 2$ ft

$x + 11$ Subst $x = 6$.

$6 + 11$

$= 17$ ft

(13) Two frames are needed with the same perimeter, one the shape of a square and one an equilateral Δ. Each side of the triangle is 6" longer than each side of the square. Find the dimensions of each frame. sq. 18", Δ. 24"



$4x = 3(x+6)$

$x = 18"$ $y = 24"$

(14) The number of deaths caused by tornados decreased 59.2% from the 1950's to the 1990's. There were 579 deaths from tornados in the 1990's. How many deaths from tornados were there in 1950's, round to nearest whole 1419

$x - .592x = 579$ (1950, 1950)

$x = \frac{579}{1 - .592}$ $1419.1 \approx$ $\boxed{1419 \text{ tornados in 1990s}}$

(15)

add a %

1.6.25 In 2006, the population of the country was 32.6 million. This represented an increase in population of 4.5% since 2001. What was the population of the country in 2001? Round to the nearest hundredth of a million.

In 2001, the population of the country was ☐ million
(Round to the nearest hundredth.)

2001 pop + % of 2001 pop = 2006 pop

= 32.6 million

$x + .045x$

$1x + 0.045x = 32.6$ million combine like terms

$1.045x = 32.6$ million isolate x

$x = \frac{32.6}{1.045}$

$x = 31.196\,17225$

↑ hundredth of a million

$x =$ **32.20 million people**

1 million = 1,000,000

32.20 million = **32,200,000 people**

Math 70 1.7

A summary of the three types of percent problems is on the front of GC-9.

If you need a complete refresher, the Math 45 lesson on these percent problems is at the end of this packet.

Basic Percent $a = p \cdot b$

a = new amount
p = percentage, written as a decimal
b = base amount

Can also be done by proportion

$$\frac{p}{100} = \frac{a}{b}$$

a = new amount
p = percentage, written as percent
b = base amount

Percent Increase and Percent Decrease

$b + pb = a$ increase

$b - pb = a$ decrease

must be the same, b = original base earliest (in time)
p = percent written as decimal
a = new amount later (in time)

Math 70 1.7

Percent Increase problem types

1) Sales tax
2) pay raise
3) investment gain
4) mark up
   (wholesale to retail price)
5) population increase
6) inflation
7) tips
   (restaurant servers)
8) "percent increase" in wording

Percent Decrease problem types

1) pay cut
2) investment loss
3) discount or sale price
   (retail price lowered)
4) population decrease
5) "percent decrease" in wording

Detailed examples in the Math 45 lecture 2.6 at the end of this lecture

Math 45 SSM 2/e 2.6 problems with Percent — Day 2

Objectives 1) Apply the basic percent equation

Day 2 { 2) Apply the percent increase equation
3) Apply the percent decrease equation.

Recall: Percent equation $a = pb$ means "a is p percent of b."

Percent increase means two steps in a single new formula

1. Find p percent of b.
2. Add result to b.

$\text{new amount} = b + p \cdot b$ OR $\text{new amt} = (1+p)b$

* CAUTION* b must be the old amount, or the amount that was at an earlier time.
Must not confuse b and new amount or you'll get the wrong answer

Types of Problems which are percent increase:

1) Sales Tax
2) Pay Raise
3) Investment gain
4) Mark up (wholesale to retail)
5) Population increase
6) Inflation
7) Tips
8) "Percent Increase"

Examples

① The sales tax rate is 5.75%. The total cost of a used Honda Civic, including sales tax, is $8460. Find the cost before tax.

$\begin{cases} \text{new amount} = 8460 \\ p = .0575 \\ b = \text{unknown} \end{cases}$ Identify values

Write formula: Sales tax is Percent Increase

$\text{new amount} = b + pb$

Substitute

$8460 = b + .0575b$

Combine like terms: $b = 1b$

$8460 = 1.0575b$

$\frac{8460}{1.0575} = b$

$b = \$8000$

CAUTION: COMMON MISTAKE:

You cannot avoid the algebra by doing only a basic percent:

$a = p \cdot b$

$a = (.0575)(8460)$

$a = 486.45$

$8460 - 486.45 = \$7973.55$ ← WRONG ANSWER. This is too small, since it's subtracting 5.75% of 8460, not 5.75% of 8000.

② A college bookstore marks up 35% over publisher's wholesale price. A textbook sells for $120. How much did it cost the bookstore?

Mark-up ⇒ Percent increase

$new = b + pb$

$\begin{cases} new = 120 \\ p = .35 \\ b = \text{unknown} \end{cases}$

Write formula
Identify values

$120 = b + .35b$
$120 = 1.35b$
$\frac{120}{1.35} = b$
$b = \$88.89$

Substitute
Combine like terms
Isolate variable

*** CAUTION *** Always round $ answers to two decimal places, even when it doesn't say to round.

Percent Decrease also means 2 steps in a single formula

1. Find p percent of b.
2. Subtract result from b.

$\text{new amount} = b - pb$ or $\text{new amt} = (1-p)b$

SAME CAUTION b must be the old amount or the amount that was at an earlier time.
Must not confuse b and the new level or you'll get the wrong answer.

Types of problems that are percent decrease:

1) Pay cut
2) Investment loss
3) Discount pricing (reg price to sale price)
4) Population decrease
5) "Percent Decrease"

③ After losing 9% of ~~the~~ investment, the account held $22,750. What was the original investment?

Investment loss ⇒ Percent decrease

$\begin{cases} \text{new amount} = 22750 \\ p = .09 \\ b = \text{unknown} \end{cases}$

Identify values

ex ③ cont

$\text{new amt} = b - p \cdot b$ — Write formula

$22750 = b - .09b$ — Substitute

$22750 = (1 - .09)b$ — Combine like terms

$22750 = .91b$

$\frac{22750}{.91} = b$ — Isolate variable

$\boxed{b = \$25000}$ — Write answer with units

④ A suit discounted 30% has a sale price of $399. What was the original price?

Sale price ⇒ percent decrease

new amt = 399
p = .3
b = unknown — Write values

$\text{new amt} = b - .3b$ — Write formula

$399 = .7b$ — Substitute, Combine like terms

$\frac{399}{.7} = b$ — Isolate variable

$\boxed{b = \$570}$ — Write answer with units.

– Mixed Practice – See handout

Math 45 Section 2.6 Direct Translation Problems Involving Percent

Problems:

H-1. A real estate commission is 3% of the selling price of $123,000. How much is the commission?

Commission = (.03)(123,000)
= $3690

Percent Equation

$a = p \cdot b$
$a = 123000$
$p = 3\% = .03$
$b =$ unknown

H-2. Your exam score is a percentage of total points available. You earned 128 points out of 160 points. What percentage did you get?

$\frac{128}{160} =$ 80%

Percent Equation

$a = p \cdot b$
$a = 128$
$b = 160$
p unknown
$128 = p \cdot 160$
divide both by 160.

H-3. You earn a 5% pay raise. Your new hourly rate is $23.70. What was your previous hourly rate?

23.70 = old rate + (.05) old rate
$x + .05x = 23.7$
$1.05x = 23.7$
$x = 22.571428...$
$x \approx$ $22.57

Percent Increase

new = b + pb
$23.70 = b + .05b$

* CAUTION *
If the problem's answer is money, you should round to the nearest hundredth even if the instructions do not say to round

H-4. Your boss gives everyone a 10% pay cut. Your previous salary was $120,000. What is your new salary?

new = old − .10(old)
new = 120,000 − (.10)(120,000)
= $108,000

Percent Decrease

new = b − p · b
new = unknown
p = .1
b = 120,000

H-5. A store marks up the wholesale price of all clothes by 35%. If the selling price of a shirt is $56, what was the wholesale price?

Selling price = wholesale + .35(wholesale)
$56 = x + .35x$
$56 = 1.35x$
$x = 41.481\overline{481}...$
$x \approx$ $41.48

Percent Increase

new = b + pb
new = 56
p = .35
b = unknown

Extra Practice

⑤ House value increased 4% over last year, now worth $205,000. What was the value last year?

Percent increase

new = b + pb

$\begin{cases} new = 208{,}000 \\ p = .04 \\ b = \text{unknown} \end{cases}$

$208{,}000 = b + .04b$

$208{,}000 = 1.04b$

$\frac{208{,}000}{1.04} = b$

$\boxed{b = \$200{,}000}$

⑥ Population of Carlsbad NM decreased from 28,000 to 25,000. Find the % decrease to nearest tenth of a percent.

Percent decrease

new = b − pb — Write formula

$\begin{cases} new = 25000 \\ b = 28{,}000 \\ p = \text{unknown} \end{cases}$ — Identify values

$25000 = 28000 - p(28000)$ — Substitute

$25000 = 28000 - 28000p$ — Isolate p.

$-28000 \quad -28000$ — Subtract 28000 both sides

$-3000 = -28000p$

$\frac{-3000}{-28000} = p$

$p = \frac{3}{28} = .107142857143$

$p \approx .107$

$\boxed{p = 10.7\%}$

⑦ You bought a tractor last year for $3750 and sell it this year for $3900. What is the rate of inflation?

inflation ⇒ percent increase

new = b + pb — Write formula

$\begin{cases} new = 3900 \\ b = 3750 \end{cases}$ p unknown — Identify values

Extra Practice ⑦ cont

$3900 = 3750 + p \cdot 3750$ Substitute

$3900 = 3750 + 3750p$ rewrite

$-3750 \quad -3750$ Subtract 3750

$150 = 3750p$

$\frac{150}{3750} = p$ Divide by 3750 } Isolate variable

$p = .04$

$\boxed{p = 4\%}$ Convert decimal to percent.

⑧ A 40% sale is advertised. A coat you've been watching is $75 originally. What is the sale price?

Sale price ⇒ Percent decrease

$new = b - pb$ Write formula

$new =$ unknown
$b = 75$
$p = .4$ } Find values

$x = 75 - .4(75)$

$\boxed{x = \$45}$ Calculate.

~ BONUS FEATURE ~

How to solve percent increase and percent decrease problems using a percent proportion.

*** CAUTION ***

You must memorize the lists of types of problems which are increase or decrease no matter which method you use.

Example ① $new = 8460$
$p = 575\%$
$b =$ unknown } percent increase

Step 1: Identify values as before, but write p as a percent, (not a decimal).

Step 2: For percent increase add $100\% + p \Rightarrow 100 + 5.75$
For percent decrease subtract $100\% - p \quad = 105.75\%$

step 3: Write a proportion

$$\frac{\text{new}}{b} = \frac{[\text{result from step 2}]}{100}$$

step 4: Substitute values

$$\frac{8460}{b} = \frac{105.75}{100}$$

step 5: Cross-multiply.

$$(8460)(100) = 105.75\,b$$

$$846000 = 105.75\,b$$

step 6: Isolate variable using algebra

$$\frac{846000}{105.75} = b$$

$$\boxed{b = \$8000}$$

Note: Either method, used correctly, should get the same answer!

② mark up ⇒ percent increase

$\begin{cases} \text{new} = 120 \\ p = 35\% \\ b = \text{unknown} \end{cases}$ Identify values.

$100\% + 35\% = 135\%$ Add 100%

$\frac{\text{new}}{b} = \frac{p}{100}$ Write proportion

$\frac{120}{x} = \frac{135}{100}$ Substitute

Yes! You could also reduce $\frac{135}{100}$ first!

$12000 = 135x$ cross-multiply

$\frac{12000}{135} = x$

$\boxed{x = \$88.89}$ round as before.

 BONUS — Proportions

③ Investment loss ⇒ percent decrease

$\begin{cases} \text{new} = 22750 \\ p = 9\% \\ b = \text{unknown} \end{cases}$ identify values

$100\% - 9\% = 91\%$ Subtract from 100% for percent decrease

$\frac{\text{new}}{b} = \frac{91\%}{100\%}$ substitute

$\frac{22750}{x} = \frac{91}{100}$

$2275000 = 91x$ cross multiply

$\frac{2275000}{91} = x$

$\boxed{x = \$25000}$

④ discount price ⇒ percent decrease

$\begin{cases} \text{new} = 399 \\ p = 30\% \\ b = \text{unknown} \end{cases}$

$100\% - 30\% = 70\%$

$\frac{\text{new}}{b} = \frac{70\%}{100\%}$

$\frac{399}{x} = \frac{70}{100}$

$39900 = 70x$

$x = \frac{39900}{70}$

$\boxed{x = \$570}$

Note: You can often choose to reduce fractions in percent proportions to get smaller numbers:

$\frac{399}{x} = \frac{7}{10}$

$3990 = 7x$

$x = \frac{3990}{7} = \$570$

— Handout — By proportions

H-① commission ⇒ basic percent

$\frac{a}{b} = \frac{p}{100}$ write basic % proportion

$\begin{cases} a = \text{unknown} \\ p = 3\% \\ b = 123{,}000 \end{cases}$ Find values

$\frac{x}{123{,}000} = \frac{3}{100}$ substitute

BONUS – Proportions

H-① – handout cont

$100x = 369000$ cross-multiply

$\frac{x = 369000}{100}$ isolate variable

$\boxed{x = \$3690}$

H-② exam score ⇒ basic percent

$\begin{cases} a = 128 \\ b = 160 \\ p = \text{unknown} \end{cases}$

$\frac{a}{b} = \frac{p}{100}$

$\frac{128}{160} = \frac{p}{100}$ ← or reduce! $\frac{4}{5} = \frac{p}{100}$

$12800 = 160p$

$\frac{12800}{160} = p$

$400 = 5p$

$\frac{400}{5} = p$

$\boxed{p = 80\%}$

Note: When using the percent proportion, the percents go in and come out as %, not decimal

H-③ pay raise ⇒ percent increase

$\begin{cases} \text{new} = 23.70 \\ p = 5\% \\ b = \text{unknown} \end{cases}$

$100\% + 5\% = 105\%$

$\frac{\text{new}}{b} = \frac{105}{100}$

$\frac{23.70}{x} = \frac{105}{100}$ ← or reduce! $\frac{23.70}{x} = \frac{21}{20}$

$2370 = 105x$

$x = \frac{2370}{105}$

$474 = 21x$

$x = \frac{474}{21}$

$\boxed{x = \$22.57}$

H-④ pay cut ⇒ percent decrease

$\begin{cases} \text{new} = \text{unknown} \\ p = 10\% \\ b = 120{,}000 \end{cases}$

$100\% - 10\% = 90\%$

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handout ④ con't

$\frac{new}{b} = \frac{90}{100}$

$\frac{x}{120{,}000} = \frac{90}{100}$

$100x = 10800000$

$x = \frac{10800000}{100}$

$\boxed{x = \$108{,}000}$

or reduce!

$\frac{x}{120{,}000} = \frac{9}{10}$

$10x = 1080000$

$x = \frac{1080000}{10}$

$\boxed{x = \$108{,}000}$

H- ⑤ mark up ⇒ percent increase

$\begin{cases} new = 56 \\ p = 35\% \\ b = \text{unknown} \end{cases}$

$100\% + 35\% = 135\%$

$\frac{new}{b} = \frac{135}{100}$

$\frac{56}{x} = \frac{135}{100}$

$5600 = 135x$

$x = \frac{5600}{135}$

$\boxed{x = \$41.48}$

or reduce!

$\frac{56}{x} = \frac{27}{20}$

$1120 = 27x$

$x = \frac{1120}{27}$

$\boxed{x = \$41.48}$

Extra Practice — by proportions

⑤ percent increase

$\begin{cases} new = 208{,}000 \\ p = 4\% \\ b = \text{unknown} \end{cases}$

$100\% + 4\% = 104\%$

$\frac{new}{b} = \frac{104}{100}$

$\frac{208000}{x} = \frac{104}{100}$

$20800000 = 104x$

$x = \frac{20800000}{104}$

$\boxed{x = \$200{,}000}$

or reduce!

$\frac{208000}{x} = \frac{26}{25}$

$5200000 = 26x$

$x = \frac{5200000}{26}$

$\boxed{x = \$200{,}000}$

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⑥ percent decrease

$\begin{cases} \text{new} = 25000 \\ p = \text{unknown} \\ b = 28000 \end{cases}$

$100\% - x = 100 - x$ ← uh-oh! Need an expression!

$\frac{\text{new}}{b} = \frac{100-x}{100}$ ← Substitute the expression

$\frac{25000}{28000} = \frac{100-x}{x}$ cross multiply

$25000x = (100-x)\,28000$

$25000x = 2800000 - 28000x$ distribute

$+28000x \qquad +28000x$

isolate variable → collect x first

$53000x = 2800000$

$x = \frac{2800000}{53000}$

$\boxed{x = 10.7\%}$

⑦ inflation ⇒ percent increase

$\begin{cases} \text{new} = 3900 \\ p = \text{unknown} \\ b = 3750 \end{cases}$

$100 + x$ ← Need an expression!

$\frac{\text{new}}{b} = \frac{100+x}{100}$

$\frac{3900}{3750} = \frac{100+x}{100}$

$390000 = 3750(100+x)$

$390000 = 375000 + 3750x$

$-375000 \qquad -375000$

$15000 = 3750x$

$\boxed{x = 4\%}$

⑧ sale price ⇒ percent decrease

new = unknown

$b = 75$

$p = 40\%$

$100\% - 40\% = 60\%$

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⑧ cont

$$\frac{\text{new}}{\text{b}} = \frac{60\%}{100\%}$$

$$\frac{x}{75} = \frac{60}{100}$$

$$100x = 4500$$

$$x = \frac{4500}{100}$$

$$\boxed{x = \$45}$$

or reduce!

$$\frac{x}{75} = \frac{3}{5}$$

$$5x = 225$$

$$x = \frac{225}{5}$$

$$\boxed{x = \$45}$$